Consistent ADD scenario with stabilized extra dimension

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Abstract

A model with one compact extra dimension and a scalar field of Brans-Dicke type in the bulk is discussed. It describes two branes with non-zero tension embedded into the space-time with flat background. This setup allows one to use a very simple method for stabilization of the size of extra dimension. It appears that the four-dimensional Planck mass is expressed only through parameters of the scalar field potentials on the branes.

1 Introduction

Nowadays models with extra space-time dimensions are widely discussed in the literature. A lot of problems are addressed with the help of such models, in particular, the hierarchy problem. In the latter case the most known models are the ADD scenario [1, 2] and the Randall-Sundrum model with two branes (usually abbreviated as RS1 model) [3] (both with compact extra space-like dimensions). The ADD scenario seems to be simpler, but the brane tension is not taken into consideration in this model. This problem was solved within the framework of the RS1 model, nevertheless this model appeared to be much more complicated. Both models were widely discussed in the literature, see reviews [4, 5].

Here we propose a model, which unifies some features of the models mentioned above and can be treated as a "consistent ADD". It describes branes with tension in the five-dimensional flat bulk and allows one to use simple method for stabilization of extra dimension's size.

2 Description of the model

Thus, we consider a space-time with one compact extra space-like dimension. Let us denote the coordinates by $\{x^M\} \equiv \{x^\mu, y\}$, $M = 0,1,2,3,4$, $\mu = 0,1,2,3$, the coordinate $x^4 \equiv y$ parameterizing the fifth dimension. As in the RS1 model, it forms the orbifold S^1/Z_2, which is realised as the circle of the circumference $2L$ with points y and $-y$ identified. It is evident that the metric g_{MN} satisfies the corresponding orbifold symmetry conditions. The branes are located at the fixed points of the orbifold, $y = 0$ and $y = L$.

The action of the model is chosen to be

$$\begin{aligned} S &= \int \Phi(x,y) R \sqrt{-g}\, d^4x dy + \\ &+ \int_{y=0} \left(\lambda_1 - \gamma_1 \left[\Phi^2(x,0) - v_1^2\right]^2\right) \sqrt{-\tilde{g}}\, d^4x + \\ &+ \int_{y=L} \left(\lambda_2 - \gamma_2 \left[\Phi^2(x,L) - v_2^2\right]^2\right) \sqrt{-\tilde{g}}\, d^4x \qquad (1) \end{aligned}$$

where $\Phi(x, y)$ is a five-dimensional scalar field (such that $\Phi(x, -y) = \Phi(x, y)$), parameters λ_1, λ_2, γ_1, γ_2, v_1 and v_2 describe the scalar field potentials on the branes (or simply the brane tensions), $\tilde{g}_{\mu\nu}$ is the induced metric on the branes. The first term in (1) corresponds to the five-dimensional Brans-Dicke theory with the Brans-Dicke parameter equal to zero, whereas the last two terms correspond to the contribution of the branes, where the Higgs-like potentials for the field $\Phi(x, y)$ are added (surely one can choose another type of potentials). It is necessary to note that the five-dimensional theory with Brans-Dicke field is widely discussed with relation to cosmology, see [6, 7] and references therein.

The equations, following from action (1), have the form (see, for example, [8])

$$\Phi\left(R_{MN} - \frac{1}{2}g_{MN}R\right) - \left(\nabla_M\nabla_N\Phi - g_{MN}g^{AB}\nabla_A\nabla_B\Phi\right) - \delta^\mu_M\delta^\nu_N \times$$

$$\times \quad \left(\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}}\tilde{g}_{\mu\nu}\frac{\lambda_1 - \gamma_1\left[\Phi^2 - v_1^2\right]^2}{2}\,\delta(y) + \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}}\tilde{g}_{\mu\nu}\frac{\lambda_2 - \gamma_2\left[\Phi^2 - v_2^2\right]^2}{2}\,\delta(y - L)\right) = 0 \quad (2)$$

(the Einstein equations), where ∇_M is the covariant derivative with respect to the metric g_{MN}, and

$$R - \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}}4\gamma_1\Phi\left(\Phi^2 - v_1^2\right)\,\delta(y) - \frac{\sqrt{-\tilde{g}}}{\sqrt{-g}}4\gamma_2\Phi\left(\Phi^2 - v_2^2\right)\,\delta(y - L) = 0 \quad (3)$$

(the equation for the scalar field).

We are going to find a solution of equations (2), (3) with $g_{MN} = diag(-1, 1, 1, 1, 1)$, i.e. with the flat five-dimensional background metric. Let us suppose that background solution for the field $\Phi(x, y)$ does not depend on the four-dimensional coordinates x. In this case the only non-trivial component of (2) takes the form

$$g_{\mu\nu}\Phi'' - \left(\tilde{g}_{\mu\nu}\frac{\lambda_1}{2}\,\delta(y) + \tilde{g}_{\mu\nu}\frac{\lambda_2}{2}\,\delta(y - L)\right) = 0, \quad (4)$$

where $\Phi'' = \frac{d^2\Phi}{dy^2}$ and equation (3) is taken into account. If

$$\lambda_1 = -\lambda_2 = \lambda, \quad (5)$$

we get

$$\Phi(y) = \frac{\lambda}{4}|y| + C, \quad (6)$$

where C is a constant and the orbifold symmetry conditions are taken into account. From equation (3) one can get

$$\Phi(y) = \Phi_0 = \frac{\lambda}{4}|y| + v_1, \quad (7)$$

$$L = \frac{4}{\lambda}\left(v_2 - v_1\right). \quad (8)$$

We see that the size L of extra dimension is stabilized (by equation (3)). It is necessary to note that in the case $\lambda > 0$ physically relevant result can be obtained only if $v_2 > v_1 > 0$.

Now let us find effective four-dimensional Planck mass on the branes. To this end it is necessary to obtain the wave-function of the massless (from the four-dimensional point of view) tensor mode of $h_{\mu\nu}$, which is the fluctuation of $\mu\nu$-component of the metric. Linearizing $\mu\nu$-component of equation (2) and dropping the scalar degrees of freedom, one can get the following equation (in the transverse-traceless gauge for the field $h_{\mu\nu}$):

$$\Phi_0 \Box h_{\mu\nu} + \Phi_0 h''_{\mu\nu} + \Phi'_0 h'_{\mu\nu} = 0. \tag{9}$$

The third term of this equation comes from the term $\nabla_\mu \nabla_\nu \Phi$ in (2). Here $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. The field $h_{\mu\nu}$ can be represented as a sum

$$h_{\mu\nu}(x,y) = \sum_n h^n_{\mu\nu}(x)\Psi_n(y), \tag{10}$$

where $\Box h^n_{\mu\nu} = m_n^2 h^n_{\mu\nu}$ and $\Psi_n(y)$ are the wave functions which correspond to the four-dimensional masses m_n. Equation (9) takes the form

$$\Phi_0 m_n^2 \Psi_n + \Phi_0 \Psi''_n + \Phi'_0 \Psi'_n = 0, \tag{11}$$

which is equivalent to

$$\Phi_0 m_n^2 \Psi_n + (\Phi_0 \Psi'_n)' = 0, \tag{12}$$

where the differential operator has the self-adjoint form. It is not difficult to show that

$$\int_{-L}^{L} \Phi_0(y)\Psi_n(y)\Psi_k(y) \sim \delta_{nk}, \tag{13}$$

i.e. the eigenfunctions of different modes are orthogonal. It is evident that $\Psi_0(y) = const$. Other eigenfunctions of equation (11) have the form

$$\Psi_n(y) = C_1 J_0\left(m_n\left[|y| + \frac{4v_1}{\lambda}\right]\right) + C_2 N_0\left(m_n\left[|y| + \frac{4v_1}{\lambda}\right]\right), \tag{14}$$

where C_1 and C_2 are constants, J_0 and N_0 are Bessel and Neumann functions. Substituting (14) into (11) and collecting terms with delta-functions (coming from $|y|$), one can get the conditions on the "boundaries" $y=0$ and $y=L$, which define the mass spectrum of the theory and the relation between C_1 and C_2 and look like

$$\Psi'_n(0) = 0, \tag{15}$$
$$\Psi'_n(L) = 0. \tag{16}$$

The equation for the mass spectrum has the form

$$N_1\left(m_n\frac{4v_1}{\lambda}\right) J_1\left(m_n\frac{4v_2}{\lambda}\right) - N_1\left(m_n\frac{4v_2}{\lambda}\right) J_1\left(m_n\frac{4v_1}{\lambda}\right) = 0. \tag{17}$$

Thus we are ready to find the relationship between the four-dimensional Planck mass and the parameters of the theory. Taking the first term of equation (1), retaining only the zero mode in (10) and using the fact that $\Psi_0(y) = const$, we can formally get

$$\int \Phi(x,y) R\sqrt{-g}\, d^4x dy \supset \int_{-L}^{L}\left(\frac{\lambda}{4}|y| + v_1\right) dy \int R_{(4)}\sqrt{-g}\, d^4x \tag{18}$$

and

$$M_{Pl}^2 = \int_{-L}^{L} \left(\frac{\lambda}{4} |y| + v_1 \right) dy = \frac{\lambda}{4} L^2 + 2 v_1 L. \tag{19}$$

Using (8) we get

$$M_{Pl}^2 = \frac{4}{\lambda} \left(v_2^2 - v_1^2 \right), \tag{20}$$

i.e. the four-dimensional Planck mass is defined by parameters λ, v_1 and v_2 of the brane tensions. Taking suitable values of v_1, v_2 and λ we can solve the hierarchy problem in the same way as in the ADD model with two extra dimensions. For example, one can take $M = \sqrt[4]{\lambda} \sim 100 TeV$ and $L \sim 1 eV^{-1}$ to get $M_{Pl} \simeq 10^{19} GeV$ (compare with physically interesting values of parameters in the ADD model with two extra dimensions, where five-dimensional Planck mass is chosen to be of the order of $30 TeV$, whereas the size of the extra dimension is chosen to be of the order of $10^{-2} eV^{-1}$, see [4]). If we suppose that $v_1 \sim v_2$ (i.e. they are of the same order), then $M_{v_i} \sim \sqrt[3]{v_i}$ (where $i = 1, 2$) should be of the order of $10^6 - 10^7 TeV$. It is evident, that this difference between the energy scales M and M_{v_i} appears because of the new hierarchy between M and L^{-1}. One can see that this hierarchy is inherent to the original ADD scenario with two extra dimensions too (see also [4]). Of course, the simplest way to remove hierarchy between M and M_{v_i} is to choose M and M_{v_i} ($i = 1, 2$) to be of the same order, which results in $L^{-1} \sim M \sim M_{v_i} \sim M_{Pl} \simeq 10^{16} TeV$. We can also leave M in the TeV range and fine-tune parameters v_1 and v_2 to make $L^{-1} \simeq 10^{16} TeV$ (in this case $M_{v_i} \sim 10^{16} TeV$ also). But even if only one fundamental parameter of the model is of the order of M_{Pl}, we get the same problem as in ordinary four-dimensional theory of gravity – we should explain its unnaturally large value. In the latter case one should also explain such extreme fine-tuning between v_1 and v_2. Another possibility to remove this hierarchy is to leave M and M_{v_i} (energy scales of fundamental parameters of the theory) in the TeV range, but to fine-tune parameters v_1 and v_2 to make the size L extremely large ($L \sim \frac{M_{Pl}^2}{M^3}$), like in the ADD model with one extra dimension. It is evident that such size of the extra dimension contradicts observable data.

At the same time the hierarchy between $M \sim 100 TeV$ and $L \sim eV^{-1}$ is not so dangerous as it seems. We should take into account that the size of the extra dimension L is not a fundamental parameter of the model. Indeed, *fundamental* parameters of the model are λ and v_i, whose energy scales are $M \sim 100 TeV$ and $M_{v_i} \sim 10^6 - 10^7 TeV$, and the size L is defined by these parameters. One can see that the largest energy scales in the five-dimensional theory $M_{v_i} \sim 10^6 - 10^7 TeV$ are much closer to the desirable value $1 TeV$, which roughly characterizes the energy scale of the Standard Model on the brane, than the four-dimensional Planck mass $M_{Pl} \sim 10^{16} TeV$. Moreover, the difference between energy scales of fundamental parameters M, M_{v_i} is maximally 5 orders in magnitude, which is very small in comparison with the hierarchy between the energy scales in ordinary four-dimensional theory of gravity. We need not to fine-tune parameters v_1 and v_2 – they should be of the same order only. As for the size of the extra dimension, its large value appears naturally from (8) and this situation can be interesting in view of top-table experiments for testing gravity at sub-millimeter scales.

Now let us discuss how the model can be interpreted. One can see that at the first sight the model under consideration (with only one extra dimension) manifests itself through (19) as the ADD model with two extra dimensions (because of the factor L^2). At the same time using (8) we can rewrite (20) as

$$M_{Pl}^2 = L \left(v_1 + v_2 \right), \tag{21}$$

which now represents the model as the ADD model with one extra dimension. Since λ and v_1, v_2 are all fundamental parameters, a discrepancy arises. The answer is that since the size of the extra dimension L is not a fundamental parameter of the theory, as it was noted above, both interpretations do not correspond to the real properties of the model, which is not exactly the ADD model, and therefore should be rejected.

In the end of this section let us compare the setup discussed above with some well-known multidimensional models, in which the scalar field interacting with gravity is used for stabilization of the extra dimension. The most known model is the one proposed in [9]. But the background solution found in [9] is not an exact one – backreaction of the scalar field on the metric is not taken into account in this model. At the same time the background solution found above is exact, i.e. it satisfies all equation of motion – Einstein equations and equation for the scalar field. Thus, it is more reasonable to compare our setup with the one proposed in [10], where an exact solution for gravity and scalar field in five dimensions was obtained (the so-called stabilized Randall-Sundrum model). It is evident that consistent comparison can be made only if we transform (1) to the Einstein frame (it can be made with the help of a conformal rescaling). The bulk part of rescaled action describes scalar field minimally coupled to five-dimensional gravity. It differs from the bulk action proposed in [10] in the absence of the bulk potential. Thus, it is evident that the models have different background solutions. Nevertheless, methods of stabilization utilized in these models are similar – the size of extra dimension is defined by the boundary conditions on the branes and depends on the parameters of scalar potentials on the branes, contrary to the case of [9], where the size of extra dimension is defined by minimization of the effective four-dimensional scalar potential (which can be obtained from the averaged five-dimensional action). In this connection it is necessary to mention paper [11], in which solution for the system of bulk scalar field without bulk potential minimally coupled to five-dimensional gravity was found. Stabilization of the extra dimension is also achieved by the boundary conditions on the branes. Because of the absence of the bulk scalar field potential this model seems to be similar in some sense to the model discussed in this paper. At the same time the solution found in [11] describes warped brane world and the dynamics of the scalar field comes from the kinetic term, contrary to our case with flat bulk and dynamics for the scalar field coming from non-minimal interaction of this field with five-dimensional curvature. But since both models are similar with respect to the absence of the bulk scalar field potential, it would be interesting to compare predictions of these models in equal frames.

As for the frame which is chosen for action (1), the background solution in the Jordan frame seems to have the simplest form and, as it was noted before, it has some "intersections" with the ADD model. In addition, the terms in (1) corresponding to brane tension have classical form in this frame. Moreover, since we do not know which frame is the "real" one, there are no any strong objections against choosing Jordan one.

3 Scalar sector

Now let us make a brief investigation of the scalar sector of the theory. To this end we need linearized equations of motion for different components of metric fluctuations and for the scalar field. Let us denote the fluctuations of metric by h_{MN}, where $M, N = 0, .., 4$, and fluctuation

of the field Φ by φ. Thus

$$g_{MN}(x,y) = \eta_{MN} + h_{MN}(x,y),$$
$$\Phi(x,y) = \Phi_0 + \varphi(x,y),$$

where η_{MN} is the flat five-dimensional metric. The corresponding gauge transformations look like

$$h^{(1)}_{MN} = h_{MN} - \partial_M \xi_N - \partial_N \xi_M, \tag{22}$$
$$\varphi^{(1)} = \varphi - \Phi_0' \xi_4. \tag{23}$$

It is not difficult to show, that one can impose the following gauge on the fields $h_{\mu 4}$, h_{44} and φ:

$$h_{\mu 4} = 0, \tag{24}$$
$$\varphi(x,y) - \frac{1}{2}\,\Phi_0(y) h_{44}(x,y) = f(x), \tag{25}$$

i.e. the new field f depends only on four-dimensional coordinates. Indeed, the corresponding transformations look like

$$\frac{\varphi^{(1)}(x,y)}{\Phi_0^2} - \frac{h^{(1)}_{44}(x,y)}{2\Phi_0} = \frac{\varphi(x,y)}{\Phi_0^2} - \frac{h_{44}(x,y)}{2\Phi_0} + \left(\frac{\xi_4}{\Phi_0}\right)', \tag{26}$$

which demonstrates the possibility to impose the gauge (25) (see [12] for details).

We also note that after imposing this gauge we are left with residual gauge transformations $\xi_\mu(x,y) = \epsilon_\mu(x)$, which are responsible for determining the physical degrees of freedom of the massless four-dimensional graviton.

The corresponding equations of motion can be easily obtained from (2) and (3), and for convenience we will present here expressions for the linear approximation of the term $\nabla_M \nabla_N \Phi - g_{MN} g^{AB} \nabla_A \nabla_B \Phi$:

$$\begin{aligned} \nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} g^{AB} \nabla_A \nabla_B \Phi = \partial_\mu \partial_\nu \varphi + \frac{1}{2} h'_{\mu\nu} \Phi_0' - h_{\mu\nu} \Phi_0'' + \\ + \eta_{\mu\nu} h_{44} \Phi_0'' - \eta_{\mu\nu} \Box \varphi - \frac{1}{2} \eta_{\mu\nu} h' \Phi_0' - \eta_{\mu\nu} \varphi'' + \frac{1}{2} \eta_{\mu\nu} h'_{44} \Phi_0', \end{aligned} \tag{27}$$

$$\nabla_\mu \nabla_4 \Phi - g_{\mu 4} g^{AB} \nabla_A \nabla_B \Phi = \partial_\mu \partial_4 \varphi - \frac{1}{2} \partial_\mu h_{44} \Phi_0', \tag{28}$$

$$\nabla_4 \nabla_4 \Phi - g_{44} g^{AB} \nabla_A \nabla_B \Phi = -\Box \varphi - \frac{1}{2} h' \Phi_0'. \tag{29}$$

For our purposes we will need the equations for the $\mu 4$- and 44-components of metric fluctuations, equation for the field φ and contracted equation for the $\mu\nu$-component of metric fluctuations. Substituting representation

$$\begin{aligned} h_{\mu\nu}(x,y) &= b_{\mu\nu}(x,y) - \frac{1}{3}\eta_{\mu\nu} h_{44}(x,y) + \frac{4}{3}\frac{\partial_\mu \partial_\nu}{\Box} h_{44}(x,y) - \\ &- \frac{m_2^2}{6L}\eta_{\mu\nu}\frac{1}{\Box}\varphi(x,L) + \frac{m_2^2}{4L} y^2 \frac{\partial_\mu \partial_\nu}{\Box}\varphi(x,L) - \\ &- \frac{m_1^2}{6L}\eta_{\mu\nu}\frac{1}{\Box}\varphi(x,0) + \frac{m_1^2}{4L}(|y|-L)^2 \frac{\partial_\mu \partial_\nu}{\Box}\varphi(x,0) \end{aligned} \tag{30}$$

into these equations, we get:
1) $\mu4$-component

$$\partial_4\left(\partial^\nu b_{\mu\nu}-\partial_\mu b\right)=0, \tag{31}$$

2) 44-component

$$\Phi_0\left(\partial^\mu\partial^\nu b_{\mu\nu}-\Box b\right)-b'\Phi_0'+\frac{1}{2L}\left(v_2m_1^2\varphi(0)+v_1m_2^2\varphi(L)\right)-2\Box f=0, \tag{32}$$

3) equation for the field φ

$$\partial^\mu\partial^\nu b_{\mu\nu}-\Box b-b''=0, \tag{33}$$

where m_1 and m_2 – masses of the scalar field φ coming from the stabilizing potentials on the branes, $\varphi(0)=\varphi(x,0)$, $\varphi(L)=\varphi(x,L)$.

From equations (31), (33) it follows that $b''=0$. Using the symmetry conditions one can get $b'=0$. With the help of residual gauge transformations $\xi_\mu(x,y)=\epsilon_\mu(x)$ it is possible to impose the transverse-traceless gauge $b=0$, $\partial^\mu b_{\mu\nu}=0$ (see [12] for details). In this case contracted $\mu\nu$-equation takes the form

$$\Phi_0\left[\frac{3m_1^2}{2}\varphi(0)\delta(y)+\frac{3m_2^2}{2}\varphi(L)\delta(y-L)-\frac{m_1^2}{L}\varphi(0)-\frac{m_2^2}{L}\varphi(L)\right]- \\ -2\Phi_0\Box h_{44}-2h_{44}''\Phi_0-2h_{44}'\Phi_0'=0. \tag{34}$$

The boundary conditions can be obtained directly from this equation. Indeed, let us integrate this equation from $-\epsilon$ to ϵ and then take the limit $\epsilon\to 0$. Using the symmetry conditions and the fact that the first derivative of the fields can have a jump at the point $y=0$, we easily get the boundary (junction) condition at the point $y=0$. Analogously we get the boundary (junction) condition at the point $y=L$. Thus the result looks like

$$\left[\Phi_0\left(\frac{3m_1^2}{2}\varphi-4h_{44}'\right)\right]|_{y=+0}=0, \tag{35}$$

$$\left[\Phi_0\left(\frac{3m_2^2}{2}\varphi+4h_{44}'\right)\right]|_{y=L-0}=0. \tag{36}$$

Thus equations of motion for the scalar degrees of freedom do not decouple.

It is necessary to note that we use complete equations of motion in the whole bulk (including contributions with delta-functions coming from the branes) in the gauge in which branes remain straight and then extract junction condition at the points the branes are located at with the help of procedure described above. Then we solve equations of motion on the segment $[0,L]$ with the junction conditions playing the role of boundary conditions. Another approach is utilized, for example, in papers [13, 14], where the induced equation on the brane and corresponding junction condition (in more general form) are obtained directly using Gauss and Codazzi equations. Our approach is more convenient for studying the linearized theory in the whole bulk, not only in the vicinity of the brane, but it is more technically complicated than that utilized in [13, 14].

One can see that equation (34) and corresponding conditions (35), (36) are rather complicated. In [15] it was suggested to use the "stiff boundary potential" limit for the model proposed in [10] to simplify the analysis. It appears that this limit is very convenient in our

case too. Namely, if $\gamma_{1,2} \to \infty$ (i.e. if $\gamma_{1,2}$ are much larger than other parameters with the same dimensionality, this also means that $m_{1,2} \to \infty$) then the conditions for the field φ are just

$$\varphi(0) = \varphi(L) = 0, \tag{37}$$

i.e. the scalar field φ decouples from the theory on the branes. Thus from equations (32) and (34) we get

$$\Box f = 0, \tag{38}$$

$$\Phi_0 \Box h_{44} + \Phi_0 h''_{44} + \Phi'_0 h'_{44} = 0. \tag{39}$$

Thus (39) is analogous to equation (9) for the tensor modes. In addition we have the following extra boundary conditions for the field h_{44} coming from (25), (37) and (38):

$$\Box h_{44}|_{y=0} = \Box h_{44}|_{y=L} = 0. \tag{40}$$

Eigenvalue problem, which corresponds to equations (39) and (40), has the form

$$\Phi_0 \mu_n^2 \tilde{\Psi}_n(y) + \Phi_0 \tilde{\Psi}''_n(y) + \Phi'_0 \tilde{\Psi}'_n(y) = 0, \tag{41}$$

$$\mu_n^2 \tilde{\Psi}_n(y)|_{y=0} = \mu_n^2 \tilde{\Psi}_n(y)|_{y=L} = 0, \tag{42}$$

where $\tilde{\Psi}_n(y)$ is the wave function of the corresponding four-dimensional mode $\Box h^n_{44}(x) = \mu_n^2 h^n_{44}(x)$ with the mass μ_n. Equation (42) can be satisfied if:

1. $$\mu_n \neq 0$$

   and

   $$\tilde{\Psi}_n(0) = \tilde{\Psi}_n(L) = 0.$$

   But boundary conditions following from (41) are $\tilde{\Psi}'_n(0) = \tilde{\Psi}'_n(L) = 0$ (see (15), (16)), which are consistent with $\tilde{\Psi}_n(0) = \tilde{\Psi}_n(L) = 0$ only if $\tilde{\Psi}_n(y) \equiv 0$.

2. $$\mu_n = 0.$$

   In this case equation (41) has the following solution in the bulk

   $$\tilde{\Psi}'_n = \frac{const}{\Phi_0}.$$

   From the symmetry conditions for the field h_{44} ($h_{44}(-y) = h_{44}(y)$) it follows that $const = 0$ and $\tilde{\Psi}_n$ does not depend on the coordinate of extra dimension ($\tilde{\Psi}_n = const$). From (25) and (37) we get

   $$\Phi_0(0)\tilde{\Psi}_n(0) = \Phi_0(L)\tilde{\Psi}_n(L),$$

   which implies that $\tilde{\Psi}_n(y) \equiv 0$.

Finally we get $h_{44}(x,y) \equiv 0$. Since f does not depend on the coordinate of extra dimension, we get $f(x) \equiv 0$. Thus, the scalar sector totally drops out from the theory, and we get effective four-dimensional gravity without any extra scalars. The model appears to be stable in this case. It is evident that disappearance of scalars is simply an artifact of the "stiff boundary potential" limit used. At the same time this situation differs considerably from the case of stabilized Randall-Sundrum model [10], where the radion field survives in the case of "stiff boundary potential" (see [15, 16]). At first glance the absence of the scalar sector in the model seems to be not very dangerous – indeed, in paper [17] it was shown, that in the Randall-Sundrum model with brane-localized curvature terms the radion field is absent in the linear approximation (because of the additional symmetry), whereas the effective theory on the brane appears to be acceptable. Moreover, the absence of scalars can ensure the absence of extra effects which could be in contradiction with the experimental data (such as a very light radion). At the same time the tensor structure of the four-dimensional massless graviton seems to be incorrect in the model as it is. Indeed, the traceless-transverse conditions $b = 0$, $\partial^\mu b_{\mu\nu} = 0$ following from (33), are not in agreement with the ordinary equation for the massless graviton $\Box\left(b_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} b\right) \sim T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor of matter. In other words, if we place matter on the brane, there should appear a massless scalar field (this scalar field is not necessarily consists of the radion and/or field φ, see [17], where analogous situation arises), which is additional to the massless graviton as viewed from the point of view of a four-dimensional observer on the brane, and which interacts with the trace of the energy-momentum tensor. We can find an analogy to this situation in electrodynamics. It is a common knowledge that longitudinal photons do not appear in the asymptotic states (on the mass shell), whereas their contribution is important in the radiative corrections (off the mass shell). This "reappearing" scalar field is very similar to longitudinal photons: it is absent in the asymptotic states, but it is absolutely necessary for consistently describing the interaction off the mass shell. Thus we get the same problem as those in the DGP model [18] and in the theory of four-dimensional massive gravity [19, 20]. Maybe it is a consequence of the total absence of the scalar sector in this approximation in case of absence of matter on the branes, which "enforces" the system to create an extra field to compensate the absence of the radion and the field φ (for example, in the RS2 model the absence of the radion field leads even to contradiction [12]). As for the general case (arbitrary values of m_1, m_2 and presence of matter on the branes), its analysis is a rather complicated task. But in this connection it is necessary to note that there is a lot of possibilities to change the linearized equations of motion without breakdown of the background solution. One can add kinetic terms for the field Φ on the branes, terms describing non-minimal coupling of the field Φ to gravity on the branes and, as a matter of fact, the four-dimensional scalar curvature:

$$\int_{y=y_i} \left(-\alpha_i \partial^\mu \Phi(x,y_i)\partial_\mu \Phi(x,y_i) + \beta_i \Phi(x,y_i)\tilde{R} + \psi_i \tilde{R}\right)\sqrt{-\tilde{g}}d^4x, \tag{43}$$

where α_i, β_i and ψ_i are arbitrary constants. Such modification, inspired by the scheme proposed in [18], provides a lot of possibilities to change the spectrum of the theory considerably (and probably can improve the incorrect tensor structure of the massless graviton). Moreover, we think that such modification of original action (1) can lead to more interesting consequences than those for the original action. But its examination exceeds the limits of this paper.

4 Conclusion

In this paper a model describing the scalar field non-minimally coupled to gravity in the space-time with one extra dimension is proposed. It possesses remarkable features: branes *with tension* in the *flat* five-dimensional background; a simple and natural way of the extra dimension's size stabilization; the four-dimensional Planck mass is defined by the parameters of the scalar field potentials on the branes; the value of the size of the extra dimension appears to be interesting in view of testing gravity at sub-millimeter scales.

At the same time it is necessary to make more thorough investigation of linearized gravity on the branes in general case (arbitrary values of m_1 and m_2) in the presence of matter on one of the brane (to answer the question about tensor structure of massless graviton), as well as to obtain a second variation Lagrangian (to answer the questions about possible appearance of ghosts in the model). But these problems call for more detailed investigation.

It should be also noted that just this mechanism is applicable only in the case of one compact extra dimension. But in any case this model seems to be quite useful because it suggests a possible way for constructing models with flat background and tension-full branes.

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References

[1] Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: "The hierarchy problem and new dimensions at a millimeter", Phys. Lett. B **429**, 263 (1998)

[2] Antoniadis I., Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: "New dimensions at a millimeter to a Fermi and superstrings at a TeV", Phys. Lett. B **436**, 257 (1998)

[3] Randall, L., Sundrum, R.: "A large mass hierarchy from a small extra dimension", Phys. Rev. Lett. **83**, 3370 (1999)

[4] Rubakov, V. A.: "Large and infinite extra dimensions: an introduction", Phys. Usp. **44**, 871 (2001)

[5] Kubyshin, Yu. A.: "Models with extra dimensions and their phenomenology", arXiv:hep-ph/0111027

[6] Mendes, L. E., Mazumdar, A.: "Brans-Dicke brane cosmology", Phys. Lett. B **501** 249 (2001)

[7] Arik, M., Ciftci, D.: "Brane world cosmology in Jordan-Brans-Dicke theory", Gen. Rel. Grav. **37**, 2211 (2005)

[8] Misner, W., Thorne, K., Wheeler, J.: "Gravitation". W. H. Freeman (1973)

[9] Goldberger, W. D., Wise, M. B.: "Modulus stabilization with bulk fields", Phys. Rev. Lett. **83**, 4922 (1999)

[10] DeWolfe, O., Freedman, D. Z., Gubser, S. S., Karch, A.: "Modeling the fifth dimension with scalars and gravity", Phys. Rev. D **62**, 046008 (2000)

[11] Kanti, P., Olive, K. A., Pospelov, M.: "Static solutions for brane models with a bulk scalar field", Phys. Lett. B **481**, 386 (2000)

[12] Smolyakov, M. N., Volobuev, I. P.: "Is there the radion in the RS2 model?", Central Eur. J. Phys. **2**, 25 (2004)

[13] Shiromizu, T., Maeda, K. i., Sasaki, M.: "The Einstein equations on the 3-brane world", Phys. Rev. D **62**, 024012 (2000)

[14] Maeda, K. i., Wands, D.: "Dilaton gravity on the brane", Phys. Rev. D **62**, 124009 (2000)

[15] Csaki, C., Graesser, M. L., Kribs, G. D.: "Radion dynamics and electroweak physics", Phys. Rev. D **63**, 065002 (2001)

[16] Boos, E. E., Mikhailov, Yu. S., Smolyakov, M. N., Volobuev, I. P.: "Energy scales in a stabilized brane world", Nucl. Phys. B **717**, 19 (2005)

[17] Smolyakov, M. N.: "Brane induced gravity in warped backgrounds and the absence of the radion", Nucl. Phys. B **695**, 301 (2004)

[18] Dvali, G. R., Gabadadze, G., Porrati, M.: "4D gravity on a brane in 5D Minkowski space", Phys. Lett. B **485**, 208 (2000)

[19] van Dam, H., Veltman, M. J.: "Massive and massless Yang-Mills and gravitational fields", Nucl. Phys. B **22**, 397 (1970)

[20] Zakharov, V.I.: "Linearized gravitation theory and the graviton mass", JETP Lett. **12**, 312 (1970)